# Charles Darwin University

Final Examination

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| Family Name | |
| Given Name/s | |
| Student Number | |
| Teaching Period | Semester 1, 2018 |

| SMA101 – Mathematics 1A | DURATION | |
|---|---|---|
| | Reading Time: | 10 minutes |
| | Writing Time: | 180 minutes |

## INSTRUCTIONS TO CANDIDATES

1. Answer all **six** questions.
2. All questions are of equal value, and parts carry marks as indicated.
3. Read <u>**ALL**</u> questions carefully.
4. Show all working neatly in all parts. Answers without working details will attract little marks.
5. All symbols, unless stated otherwise, have their usual meanings.

## EXAM CONDITIONS

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| <u>You may begin writing from the commencement of the examination session.</u> The reading time indicated above is provided as a guide only. |
| This is a CLOSED BOOK examination |
| Any non-programmable calculator is permitted |
| No handwritten notes are permitted |
| No dictionaries are permitted |

| ADDITIONAL AUTHORISED MATERIALS | EXAMINATION MATERIALS TO BE SUPPLIED |
|---|---|
| No additional printed material is permitted | 1 x 20 Page Book<br>1 x Scrap Paper<br>Formula Sheet/s |

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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**Answer ALL questions in the Answer Booklet provided.**

Marks for each question are indicated.

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**Question 1**

(a) Given that $f(x) = \sqrt{4 - x^2}$, find the natural domain and the range of $f(x)$. (Marks: 4)

(b) Sketch the graph of the function:

$$y = \frac{1}{2}\ln x + 1$$

by appropriately modifying the function $y = \ln x$. (Marks: 4)

(c) Given that:

$$f(x) = \frac{1 + x}{1 - x}$$

$$g(x) = \frac{x}{1 - x}$$

Find expression of $g\ o\ f$, and state the domain of the composition.

(Marks: 6)

(d) Find the limit of the following:

$$(i)\quad \lim_{t \to 2} \frac{t^3 + 3t^2 - 12t + 4}{t^3 - 4t}$$

$$(ii)\quad \lim_{x \to -\infty} \frac{\sqrt{3x^4 + x}}{x^2 - 8}$$

(Marks: 6)

**Question 2**

(a) Solve the following equalities and in each case sketch the solution on a coordinate line:

$$(i) \qquad x^2 - 4x > 12$$

$$(ii) \qquad \frac{x}{x-3} < 4$$

(Marks: 8)

(b) A robot moves in the positive direction along a straight line so that after *t* minutes its distance is $s = 6t^4$ feet from the origin.

(i) Find the average velocity of the robot over the time interval [2,4] minutes.

(ii) Find the instantaneous velocity of the robot at *t=2* minutes. (Marks: 7)

(c) Find *dy*/*dx* of the following functions by implicit differentiation:

$$x^2y + 3xy^3 - x = 3$$

(Marks: 5)

**Question 3**

(a) Using the definition of the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

of a function $f(x)$, find $f'(x)$ of $f(x) = 6x^{-3/2}$. (Marks: 7)

(b) Find a value of the constant *k*, if any, that will make the following functions continuous:

$$f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x + k, & x > 2 \end{cases}$$

(Marks: 3)

(c) Find *dy*/*dx* of the following:

(i) $y = (5x+8)^7(1-\sqrt{x})^6$ (ii) $y = \log_{10}(\sin^2 x)$

(Marks: 5)

(d) Evaluate the following integrals:

(i) $\int (x^2 + 3x - 1)\, dx$ (ii) $\int \frac{1}{1+\sin\theta} d\theta$ (Marks: 5)

## Question 4

(a) Evaluate the following definite integrals:

(i) $\int_1^4 \frac{1}{\sqrt{x}}\,dx$ (ii) $\int_0^{\pi} \sin t\,dt$ (Marks: 5)

(b) Express the following matrix equation as a system of linear equations:

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 7 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

(Marks: 3)

(c) $A$, $B$, $C$, $D$ and $E$ are matrices of sizes $(4 \times 5)$, $(4 \times 5)$, $(5 \times 2)$, $(4 \times 2)$ and $(5 \times 4)$, respectively. Find which of the following matrix operations are defined. For those which are defined, give the size of the resulting matrix.

(i) $BA$

(ii) $AC + D$

(iii) $AE + B$

(iv) $AB + B$

(v) $E(A + B)$

(Marks: 5)

(d) Find all values of $\lambda$ for which det(A) = 0:

$$A = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$$

(Marks: 7)

**Question 5**

(a) Find the minor $M_{22}$ and cofactor $C_{22}$ of the matrix:

$$A = \begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix}$$

(Marks: 4)

(b) Solve the following linear system of equations by Gaussian elimination.

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

(Marks: 6)

(c) Find a unit vector that is orthogonal to both **u** = (1, 0, 1) and **v** = (0, 1, 1).

(Marks: 4)

(d) Find a nonzero vector **a** with terminal point *Q*(3, 0, -5) such that **a** is oppositely directed to vector **b** = (6, 7, -3). (Marks: 6)

**Question 6**

(a) Find the dot product **a.b** of the following vectors and express it in its simplest form:

(i) **a** = (2, 3) and $\mathbf{b} = (5, -7)$

(ii) $\mathbf{a} = (t, t^2)$ and $\mathbf{b} = (\cos^2 t, \frac{\sin^2 t}{t})$ .

(Marks: 5)

(b) Find Re*(z)* and Im *(z)* of:

$$z = 3e^{-\pi i}.$$

(Marks: 5)

(c) Express the following complex expressions in the form of $a + bi$:
$i(1 + 7i) - 3i(4 + 2i)$ (Marks: 5)

(d) Use de Moivre's theorem to calculate the third power of:

$$1 + i$$

(Marks: 5)

Charles Darwin University
Faculty of Technology
SMA101: Formula Sheet

# Calculus

- Equation of a straight line of slope $m$ passing through a point $(a, b)$is : $y$-$b$ = $m$($x$-$a$) .
- If $y = 10^x$ , $\log_{10} y = x$ .
- $(x \pm y)^2 = x^2 \pm 2xy + y^2$
- $a^2 - b^2 = (a+b)(a-b)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- Quadratic Eqn. $ax^2 + bx + c = 0$ has roots as $x = -\dfrac{b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$
- $\sin^2 x + \cos^2 x = 1$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mu \sin A \sin B$
- $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$
- $\dfrac{df}{dx} = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
- $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x} = 0$
- $\dfrac{dx^n}{dx} = nx^{n-1}$
- $\dfrac{d \sin x}{dx} = \cos x$
- $\dfrac{d \cos x}{dx} = -\sin x$
- Product rule: $\dfrac{df(x)g(x)}{dx} = fx)\dfrac{dg(x)}{dx} + g(x)\dfrac{df(x)}{dx}$
- Quotient rule: $\dfrac{d}{dx}\left[\dfrac{f}{g}\right] = \dfrac{g\dfrac{df}{dx} - f\dfrac{dg}{dx}}{g^2}$
- $\dfrac{df(g(x))}{dx} = \dfrac{df(g)}{dg}\dfrac{dg(x)}{dx}$ (Chain rule)

- $\frac{de^{\alpha x}}{dx} = \alpha e^{\alpha x}$
- $\frac{d\tan x}{dx} = \sec^2 x$
- Relative extremum $\frac{df(x)}{dx}$ =0
- $\frac{d\cot x}{dx} = -\csc^2 x$
- Volume of a sphere of radius $r$: $V = \frac{4}{3}\pi r^3$
- Volume of a cone of radius $r$:and height $h$ : $V = \frac{1}{3}\pi r^2 h$.
- Area of a circle of radius $r$, $A = \pi r^2$

INTEGRATION FORMULAS

| DIFFERENTIATION FORMULA | INTEGRATION FORMULA | DIFFERENTIATION FORMULA | INTEGRATION FORMULA |
|---|---|---|---|
| 1. $\frac{d}{dx}[x] = 1$ | $\int dx = x + C$ | 8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$ | $\int \csc x \cot x\, dx = -\csc x + C$ |
| 2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$ | $\int x^r\, dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$ | 9. $\frac{d}{dx}[e^x] = e^x$ | $\int e^x\, dx = e^x + C$ |
| 3. $\frac{d}{dx}[\sin x] = \cos x$ | $\int \cos x dx = \sin x + C$ | 10. $\frac{d}{dx}\left[\frac{b^x}{\ln b}\right] = b^x \quad (0 < b, b \neq 1)$ | $\int b^x\, dx = \frac{b^x}{\ln b} + C \quad (0 < b, b \neq 1)$ |
| 4. $\frac{d}{dx}[-\cos x] = \sin x$ | $\int \sin x\, dx = -\cos x + C$ | 11. $\frac{d}{dx}[\ln \lvert x \rvert] = \frac{1}{x}$ | $\int \frac{1}{x} dx = \ln\lvert x \rvert + C$ |
| 5. $\frac{d}{dx}[\tan x] = \sec^2 x$ | $\int \sec^2 x\, dx = \tan x + C$ | 12. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$ | $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$ |
| 6. $\frac{d}{dx}[-\cot x] = \csc^2 x$ | $\int \csc^2 x\, dx = -\cot x + C$ | 13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$ | $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$ |
| 7. $\frac{d}{dx}[\sec x] = \sec x \tan x$ | $\int \sec x \tan x\, dx = \sec x + C$ | 14. $\frac{d}{dx}[\sec^{-1}\lvert x \rvert] = \frac{1}{x\sqrt{x^2-1}}$ | $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}\lvert x \rvert + C$ |

# Linear Algebra

- If $A$ is an invertible matrix, then:
  $A^{-1}A = AA^{-1} = I$. For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $\quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$ gives

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

- $tr(A) = \sum_i a_{ii}$
- If $A$ is a $n \times n$ matrix, then: $\det(aA) = a^n \det(A)$
- $\det(A^{-1}) = \dfrac{1}{\det(A)}$
- $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$
- $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\hat{\eta} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
- $z = x + iy$ , $z^* = x - iy$
- $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$ : polar form.
- $zz^* = x^2 + y^2$
- $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$